Published by IOP Publishing for SISSA
Received: September 1, 2008
Accepted: November 11, 2008
Published: December 5, 2008

Hawking fluxes, W_∞ algebra and anomalies

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Abstract: We complete the analysis started in [arXiv:0804.0198] of the Hawking radiation calculated by means of anomaly techniques. We concentrate on a static radially symmetric BH, reduced to two dimensions. We compare the two methods used to derive the integrated Hawking radiation, based on the trace and diffeomorphism anomaly, respectively, and show that they can be reduced to the same basic elements. We then concentrate on higher moments of the Hawking radiation and on higher spin currents, and show that, similarly to trace anomalies, also diffeomorphism anomalies are absent from the conservation laws of higher spin currents. We show that the predictivity of the method is due to the W_∞ current algebra underlying the effective model that describes matter around the black hole.

Keywords: Anomalies in Field and String Theories, Conformal and W Symmetry, Black Holes.

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1. Introduction

Hawking radiation [1, 2] is a universal phenomenon which does not depend on the details of the collapse that gives rise to a black hole. Therefore one would expect that there exist methods to calculate it that have the same character of universality. Local anomalies have such a characteristic, because all anomalies have a universal form, only the coefficients in front of them are model dependent. A first attempt to compute Hawking radiation by exploiting trace anomalies was made long time ago by Christensen and Fulling, [49], and reproposed subsequently by [51, 52] in a modified form. More recently a renewed attention to the same problem has been pioneered by the paper [3], where diffeomorphisms anomalies have been used instead of trace anomalies. This paper is at the origin of a considerable activity with numerous contributions [4 – 10, 12 – 48].

The purpose of the present paper, which is a sequel to [11], is to assess the role of anomalies in computing the thermal spectrum of the Hawking radiation. Our conclusion is that, while anomalies (trace or diffeomorphism) can be used to compute the integrated Hawking radiation, this is not the case for higher moments. Rather we find that there exists an underlying structure at the basis of the universality of Hawking radiation: this is a W_∞ algebra which characterizes the underlying matter model describing the radiation.

In this paper, as in [11], in order to be able to discuss the essential aspects while avoiding inessential complications, we will stick to the simplest case of a static chargeless black hole with metric

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega^2 \tag{1.1}$$

$f(r)$ near the horizon behaves like $f(r) \approx 2\kappa(r - r_H)$, where κ is the surface gravity. An essential step in this kind of approach is the reduction to a two-dimensional problem. This can be done by using radial symmetry, postulating the independence of the polar coordinates θ, φ and expanding the fields in spherical harmonics. For instance, for a scalar field, $\phi(t, r, \theta, \varphi) = \sum_{lm} Y_{lm}(\theta, \varphi)\, \phi_{lm}(t, r)$. One then integrates, in the action, over the polar angles. This has been done in some details, for instance, in [6], the result being a theory of infinite many complex scalar fields ϕ_{lm} interacting with the background gravity specified by the metric

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 \tag{1.2}$$

In the following we will retain only one of all these complex scalar fields. The analysis for all the other scalar fields is the same, what is left out from our analysis is how to resum all these contributions and obtain some four-dimensional information (see however the comment at the end of section 3).

In the first part of our paper we review the two methods based on anomalies, the diff and trace anomaly method. The purpose is to stress that they are actually based on the same basic formulas and same basic requirements (no ingoing flux from infinity and vanishing of energy-momentum tensor at the horizon). Next we take up the problem of higher moments of the Hawking radiation. Following [6 − 9], we attribute these higher fluxes to phenomenological higher spin currents, i.e. higher spin generalizations of the energy-momentum tensor. In [11] it was shown that these currents can be constructed out of a W_∞ algebra. It is the properties of this W_∞ algebra that explain the higher moments of Hawking radiation. As was shown in [11] the higher spin currents are not anomalous, at variance with [8, 9], where, in a different (spinorial) matter model, anomalies were found in the conservation laws and traces of higher spin currents. In this paper we complete the analysis started in [11], where, using consistency methods, the absence of true trace anomalies was proved at least for the fourth order current. Here we deal with the far more complicated case of diff anomalies. The result is invariant: there cannot exist any true diff anomalies in the fourth order current. This confirms a well founded prejudice according to which true gravitational anomalies can exist when there is a precise correspondence between number of derivative in the anomaly polynomial and space-time dimensions.

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The conclusion of our analysis is that the universal element that explains the universal character of the Hawking fluxes lies in the W_∞ algebra underlying the matter model for radiation.

2. Review of the anomaly methods

In [3] the method used was based on the diffeomorphism anomaly in a two-dimensional effective field theory near the horizon of a radially symmetric static black hole. The basic argument is that, since just outside the horizon the ingoing modes cannot classically influence the physics outside the black hole, they can be integrated out, giving rise to an effective theory of purely outgoing modes. So the physics in that region can be described by an effective two-dimensional chiral field theory (of infinite many fields). This implies an effective breakdown of the diffeomorphism invariance. The ensuing anomaly equation can be utilized to compute the outgoing flux of radiation. The latter appears as the quantum factor that restores the diffeomorphism symmetry.

2.1 Diff anomaly method

Let us describe in detail the corresponding derivation as given, in a somewhat simplified form, in [36].[1] The range of r contains two relevant regions: the region o, defined by $r > r_H + \epsilon$, r_H being the horizon radius, and the region H, defined by $r_H < r < r_H + \epsilon$. The region H is where the ingoing modes have been integrated out, therefore the effective field theory there is anomalous, while in o we expect a fully symmetric theory. This is expressed by a vanishing energy momentum tensor covariant divergence

$$\nabla_\mu T^\mu{}_{\nu(O)} = 0, \tag{2.1}$$

while in the H region we have

$$\nabla_\mu T^\mu{}_{\nu(H)} = \frac{\hbar c_R}{96\pi} \epsilon_{\nu\mu} \partial^\mu R \tag{2.2}$$

This is the covariant form of the diffeomorphism anomaly, with a coefficient appropriate for chiral (outgoing or right) matter with central charge c_R. In (2.2) $\epsilon_{\mu\nu} = \sqrt{-g}\varepsilon_{\mu\nu}$, where ε is the numerical antisymmetric symbol ($\varepsilon_{01} = 1$). In the case of the background metric we are considering, the determinant is -1. Since the metric is also static, the two equations above take, for T^r_t, a very simple form:

$$\partial_r T^r_{t(o)} = 0 \tag{2.3}$$

and

$$\partial_r T^r_{t(H)} = \partial_r N^r_t \equiv \partial_r \left(\frac{\hbar c_R}{96\pi} \left(f f'' - \frac{1}{2}(f')^2 \right) \right) \tag{2.4}$$

[1] After completion of this paper one of the author, R.Banerjee, has pointed out to us that the diff anomaly method can be further simplified by using a single Ward identity instead of two as in the presentation below. This does not change however our conclusions in section 3.

respectively. Now we integrate these equations in the respective regions of validity

$$T^r_{t(o)} = a_o \tag{2.5}$$

and

$$T^r_{t(H)}(r) = a_H + N^r_t(r) - N^r_t(r_H) \tag{2.6}$$

We remark that a_o, being constant, determines (together with the condition that there is no ingoing flux from infinity) the outgoing energy flux. This is the quantity we would like to know. To this end we define the overall energy-momentum tensor.

$$T^r_t = T^r_{t(o)}\theta(r - r_H - \epsilon) + T^r_{t(H)}\left(1 - \theta(r - r_H - \epsilon)\right) \tag{2.7}$$

It is understood that ϵ is a small number which specifies the size of the region where the energy-momentum tensor is not conserved. If we take the divergence of (2.7), we get

$$\partial_r T^r_t = \left(a_o - a_H + N^r_t(r_H)\right)\delta(r - r_h - \epsilon) + \partial_r\left(N^r_t(r)H(r)\right) \tag{2.8}$$

where $H(r) = 1 - \theta(r - r_H - \epsilon)$. We can now define a new overall tensor

$$\hat{T}^r_t(r) = T^r_t(r) - N^r_t(r)H(r) \tag{2.9}$$

which is conserved

$$\partial_r \hat{T}^r_t = 0 \tag{2.10}$$

provided that

$$a_o - a_H + N^r_t(r_H) = 0 \tag{2.11}$$

Now, the condition that at the horizon the energy-momentum tensor vanishes, leads to $a_H = 0$ (see (2.6)). Therefore

$$a_o = N^r_t(r_H) = \frac{\hbar\kappa^2}{48\pi}c_R \tag{2.12}$$

This is the outgoing flux at infinity and coincides with the total Hawking radiation (see below) emitted by the black hole specified by the metric (1.2). We remark that $\hat{T}^r_t$ is constant everywhere.

2.2 Trace anomaly method

The method based on the trace anomaly was suggested long ago by Christensen and Fulling, [49] (see also [50]). Such a method has been reproposed in different forms in [51, 52] and, in particular, [7] and [9] (see also [11]). This approach is based on the argument that the near-horizon physics is described by a two-dimensional conformal field theory (see also [53 – 55]). Classically the trace of the matter energy momentum tensor vanishes on shell. However it is generally nonvanishing at one loop, due to the anomaly: $T^\alpha_\alpha = \frac{c}{48\pi}R$,

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where R is the background Ricci scalar. c is the total central charge of the matter system. The idea is to use this piece of information in order to compute the same constant a_o calculated with the previous method. Here we do not have to split the space in different regions, but we consider a unique region outside the horizon.

With reference to the metric (1.2) it is convenient to transform it into a conformal metric. This is done by means of the 'tortoise' coordinate r_* defined via $\frac{\partial r}{\partial r_*} = f(r)$. Next it is useful to introduce light-cone coordinates $u = t - r_*, v = t + r_*$. Let us denote by $T_{uu}(u,v)$ and $T_{vv}(u,v)$ the classically non vanishing components of the energy-momentum tensor in these new coordinates. Our black hole is now characterized by the background metric $g_{\alpha\beta} = e^{\varphi}\eta_{\alpha\beta}$, where $\varphi = \log f$. The energy-momentum tensor can be calculated by integrating the conservation equation and using the trace anomaly. The result is (see next section)

$$T_{uu}(u,v) = \frac{\hbar c_R}{24\pi}\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right) + T_{uu}^{(hol)}(u) \tag{2.13}$$

where $T_{uu}^{(hol)}$ is holomorphic, while T_{uu} is conformally covariant. Namely, under a conformal transformation $u \to \tilde{u} = f(u)(v \to \tilde{v} = g(v))$ one has

$$T_{uu}(u,v) = \left(\frac{df}{du}\right)^2 T_{\tilde{u}\tilde{u}}(\tilde{u},v) \tag{2.14}$$

Since, under a conformal transformation, $\tilde{\varphi}(\tilde{u},\tilde{v}) = \varphi(u,v) - \ln\left(\frac{df}{du}\frac{dg}{dv}\right)$, it follows that

$$T_{\tilde{u}\tilde{u}}^{(hol)}(\tilde{u}) = \left(\frac{df}{du}\right)^{-2}\left(T_{uu}^{(hol)}(u) + \frac{\hbar c_R}{24\pi}\{\tilde{u},u\}\right) \tag{2.15}$$

Regular coordinates near the horizon are the Kruskal ones, (U,V), defined by $U = -e^{-\kappa u}$ and $V = e^{\kappa v}$. Under this transformation we have

$$T_{UU}^{(hol)}(U) = \left(\frac{1}{\kappa U}\right)^2\left(T_{uu}^{(hol)}(u) + \frac{\hbar c_R}{24\pi}\{U,u\}\right) \tag{2.16}$$

Now we require the outgoing energy flux to be regular at the future horizon $U = 0$ in the Kruskal coordinate. Therefore at that point $T_{uu}^{(hol)}(u)$ is given by $\frac{c_R\kappa^2}{48\pi}$. We remark that this implies in particular that $T_{uu}(r = r_H) = 0$.

Since the background is static, $T_{uu}^{(hol)}(u)$ is constant in t and therefore also in r. Therefore at $r = \infty$ it takes the same value $\frac{\hbar c_R\kappa^2}{48\pi}$. On the other hand we can assume that at $r = \infty$ there is no incoming flux and that the background is trivial (so that the vev of $T_{uu}^{(hol)}(u)$ and $T_{uu}(u,v)$ asymptotically coincide).[2]

Therefore the asymptotic flux is

$$\langle T_t^r\rangle = \langle T_{uu}\rangle - \langle T_{vv}\rangle = \frac{\hbar\kappa^2}{48\pi}c_R \tag{2.17}$$

[2]We stress that vanishing of $\langle T_{vv}\rangle$ does not contradict the stress tensor conservation. T_{vv} has an expression similar to (2.13), with subscripts u replaced by v and c_R replaced by c_L, see (3.10) below. Since T_{vv}^{a-hol} vanishes at infinity and is conserved, it would seem at first that this leads to a contradiction with a formula similar to (2.16) for the ingoing part. We notice however that $V = 0$ is not the future horizon and no vanishing condition for the stress tensor is required there.

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This outgoing flux coincides with the constant a_o calculated above.

In summary we can say that the basic ingredients of the two methods are:

(a) in the first case the integration of the anomalous and non-anomalous conservation of the energy-momentum tensor, in the second case the integration of the energy-momentum conservation in the presence of a trace anomaly;

(b) in both cases we have the condition that the energy-momentum tensor vanishes at the horizon and there is no incoming energy flux from infinity.

What energy-momentum tensor vanishes at the horizon will be clarified below.

3. Comparison between the two methods

The generic case of a chiral two-dimensional theory with central charge c_R and c_L for the holomorphic and anti-holomorphic part, respectively, is characterized by the presence of both diffeomorphism and trace anomaly,

$$\nabla_\mu T^\mu{}_\nu = \frac{\hbar}{48\pi}\frac{c_R - c_L}{2}\,\epsilon_{\nu\mu}\,\partial^\mu R \tag{3.1}$$

and

$$T^\alpha_\alpha = \frac{\hbar}{48\pi}\,(c_R + c_L)\,R \tag{3.2}$$

Let us rewrite these equations in terms of the light-cone coordinates u and v introduced before. In this basis the nonvanishing metric elements take the form:

$$g_{uv} = \frac{1}{2}e^{\varphi} = -\epsilon_{uv}, \qquad g^{uv} = 2e^{-\varphi} = \epsilon^{uv} \tag{3.3}$$

and eq. (3.1) becomes

$$\nabla_u T_{uv} + \nabla_v T_{uu} = \frac{\hbar}{48\pi}\frac{c_R - c_L}{2}\epsilon_{uv}\partial_u R \tag{3.4}$$

$$\nabla_u T_{vv} + \nabla_v T_{uv} = -\frac{\hbar}{48\pi}\frac{c_R - c_L}{2}\epsilon_{uv}\partial_v R \tag{3.5}$$

On the other hand (3.2) becomes

$$T_{uv} = \frac{\hbar}{48\pi}\,\frac{c_R + c_L}{4}\,R\,e^{\varphi} \tag{3.6}$$

Replacing this with $R = -4\partial_u\partial_v\varphi\, e^{-\varphi}$ in (3.5), we get

$$\partial_v T_{uu} = \frac{\hbar}{24\pi}\,c_R\,\partial_v \mathcal{T}_{uu} \tag{3.7}$$

where

$$\mathcal{T}_{uu} = \partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2 \tag{3.8}$$

Integrating (3.7) we get

$$T_{uu}(u,v) = \frac{\hbar}{24\pi}\, c_R\, \mathcal{T}_{uu}(u,v) + T^{(hol)}_{uu}(u) \tag{3.9}$$

where $T^{(hol)}_{uu}$ depends only on u.

Similarly, integrating (3.4), one obtains

$$T_{vv}(u,v) = \frac{\hbar}{24\pi}\, c_L\, \mathcal{T}_{vv}(u,v) + T^{(a-hol)}_{vv}(v) \tag{3.10}$$

where $\mathcal{T}_{vv} = \partial_v^2\varphi - \frac{1}{2}(\partial_v\varphi)^2$, and $T^{(a-hol)}_{vv}$ depends only on v. The two equations (3.9) and (3.10) are our basic result. They are equivalent to the two equations (3.1) and (3.2).

In the "trace anomaly" method we have utilized eq. (3.9), required that the energy-momentum tensor be conserved and imposed the conditions (b) of the previous section. This, in particular, amounts to requiring $c_R = c_L$ in the region outside the horizon. We see now that the possibility to integrate (3.1) in the presence of (3.2) is actually insensitive to the relation between c_L and c_R.[3]

In the "diff anomaly" approach we integrated (3.1) in the near horizon region and the conserved energy-momentum divergence away from the horizon. Then we imposed vanishing of energy-momentum tensor at the horizon. It is obvious that we used again (3.9) and (3.10) in disguise.

It is actually possible to be more specific. We have already noticed that in the trace anomaly method $T_{uu}(r = r_H) = 0$. On the other hand we point out that $T^{(a-hol)}_{vv}$ is constant in r and t, for the same reason as $T^{(hol)}_{uu}$ is, and thus vanishes upon the request of no ingoing flux from infinity. It is also easy to see that, if $c_R = c_L$, $\mathcal{T}_{uu} = \mathcal{T}_{vv}$. Therefore $T^r_t = T_{uu} - T_{vv}$ is constant everywhere and equals the outgoing flux (2.17) at infinity. Therefore the T^r_t of subsection 2.2 equals $\hat{T}^r_t$ of subsection 2.1. And it is also clear that the energy-momentum tensor vanishing at the horizon in subsection 2.1 is to be compared with $T_{uu}(u,v)$ of subsection 2.2.

It was important to stress the basic role of (3.9) and (3.10) because, as we will see, when we come to higher spin currents, it is not possible to describe the higher flux moments by means of anomalies (either trace or diff), but the analogues of (3.9) and (3.10) still hold and provide the desired description.

It is worth at this point spending a few words about the validity of the results obtained with the above methods in relation to the reduction from 4 to 2 dimensions mentioned in the introduction. As pointed out there the reduction of a free massless scalar field (interacting with the background metric) into an infinite set of free massless scalar fields, is only valid near the horizon. Away from the horizon the equations of motion of these fields acquire a potential term. These potential terms therefore modify eqs. (3.1) and (3.2) and consequently (3.9) and (3.10) and account for the difference between 2 and 4 dimensions. In the literature one can find estimates of the effect of such modifications, see for instance [60]. They translate into a greybody factor that cuts the Hawking radiation at infinity calculated above by an order of magnitude.

[3] In other words we can integrate the trace anomaly even if $c_R \neq c_L$. This is clearly only a characteristic of two dimensions

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4. Higher moments of the Hawking radiation and higher spin currents

The thermal bosonic spectrum of the black hole is given by the Planck distribution

$$N(\omega) = \frac{g_*}{e^{\beta\omega} - 1} \tag{4.1}$$

where $1/\beta$ is the Hawking temperature and $\omega = |k|$, the absolute value of the momentum. g_* is the number of physical degrees of freedom in the emitted radiation. In two dimensions we can define the flux moments as follows

$$F_n = \frac{g_*}{4\pi} \int_{-\infty}^{+\infty} dk \frac{\omega\, k^{n-2}}{e^{\beta\omega} - 1}$$

They vanish for n odd, while for n even they are given by

$$F_{2n} = \frac{1}{2\pi} \int_0^\infty d\omega \omega^{2n-1} N(\omega) = g_* \frac{(-1)^{n+1}}{8\pi n} B_{2n} \kappa^{2n} \tag{4.2}$$

where B_n are the Bernoulli numbers ($B_2 = \frac{1}{6}, B_4 = -\frac{1}{30}, \ldots$). Therefore the outgoing flux (2.17) is seen to correspond to F_2 when $g_* = c_R$.

The authors of [7] posed a very interesting question: the outgoing flux ($= F_2$), corresponds to the integrated distribution; is it possible to describe the higher moments of the Hawking radiation in the same way as we described the lowest one, by means of an effective field theory and, in particular, by generalizing the above two methods? They suggested that this can be done in terms of higher tensorial currents, which play the role of the energy-momentum tensor for higher moments.

In [11] an example of such currents was constructed in terms of an elementary complex scalar field. If the underlying holomorphic currents satisfy a W_∞ algebra, the effective covariant currents were shown to describe precisely the higher moments of the Hawking radiation. Let us briefly review the construction of [11].

4.1 The W_∞ algebra

Higher spin currents are expressed in terms of a single complex bosonic field ($c = 2$) and use is made of the W_∞ algebra. To this end we go to the Euclidean and replace u, v with the complex coordinates $z, \bar{z}$.

Following [56] (see also [57−59]) the starting point is a free complex boson having the following two point functions

$$\begin{aligned} \langle \phi(z_1)\overline{\phi}(z_2)\rangle &= -\log(z_1 - z_2) \\ \langle \phi(z_1)\phi(z_2)\rangle &= 0 \\ \langle \overline{\phi}(z_1)\overline{\phi}(z_2)\rangle &= 0 \end{aligned} \tag{4.3}$$

The currents are defined by

$$j^{(s)}_{z\ldots z}(z) = B(s) \sum_{k=1}^{s-1} (-1)^k A^s_k \; :\partial^k_z \phi(z) \partial^{s-k}_z \overline{\phi}(z): \tag{4.4}$$

where

$$B(s) = \left(-\frac{i}{4}\right)^{s-2} \frac{2^{s-3}s!}{(2s-3)!!}, \qquad A^s_k = \frac{1}{s-1}\binom{s-1}{k}\binom{s-1}{s-k} \tag{4.5}$$

They satisfy a W_∞ algebra [56]. It is worth recalling that this W_∞ algebra has a unique central charge, which corresponds to the central charge of the Virasoro subalgebra. Therefore it has a unique basic cocycle, which is the cocycle appearing in the Virasoro subalgebra.

The first few currents are

$$\begin{aligned}
j^{(2)}_{zz} &= - \ {:}\partial_z\phi\partial_z\overline{\phi}{:} \qquad (4.6)\\
j^{(3)}_{zzz} &= \frac{i}{2}\left({:}\partial_z\phi\partial_z^2\overline{\phi}{:} - {:}\partial_z^2\phi\partial_z\overline{\phi}{:}\right)\\
j^{(4)}_{zzzz} &= \frac{1}{5}\left({:}\partial_z\phi\partial_z^3\overline{\phi}{:} -3\,{:}\partial_z^2\phi\partial_z^2\overline{\phi}{:} + {:}\partial_z^3\phi\partial_z\overline{\phi}{:}\right)\\
j^{(5)}_{zzzzz} &= -\frac{i}{14}\left({:}\partial_z\phi\partial_z^4\overline{\phi}{:} -6\,{:}\partial_z^2\phi\partial_z^3\overline{\phi}{:} +6\,{:}\partial_z^3\phi\partial_z^2\overline{\phi}{:} - {:}\partial_z^4\phi\partial_z\overline{\phi}{:}\right)\\
j^{(6)}_{zzzzzz} &= -\frac{1}{42}\left({:}\partial_z\phi\partial_z^5\overline{\phi}{:} -10\,{:}\partial_z^2\phi\partial_z^4\overline{\phi}{:} +20\,{:}\partial_z^3\phi\partial_z^3\overline{\phi}{:} -10\,{:}\partial_z^4\phi\partial_z^2\overline{\phi}{:} + {:}\partial_z^5\phi\partial_z\overline{\phi}{:}\right)
\end{aligned}$$

Normal ordering is defined as

$${:}\partial^n\phi\partial^m\overline{\phi}{:} = \lim_{z_2\to z_1}\left\{\partial^n_{z_1}\phi(z_1)\partial^m_{z_2}\overline{\phi}(z_2) - \partial^n_{z_1}\partial^m_{z_2}\langle\phi(z_1)\overline{\phi}(z_2)\rangle\right\} \tag{4.7}$$

As usual in the framework of conformal field theory, the operator product in the r.h.s. is understood to be radial ordered.

The current $j^{(2)}_{zz}(z) = - \ {:}\partial_z\phi(z)\partial_z\overline{\phi}(z){:}$ is proportional to the (normalized) holomorphic energy-momentum tensor of the model and, upon change of coordinates $z \to w(z)$, transforms as

$${:}\partial_z\phi\partial_z\overline{\phi}{:} = (w')^2 \ {:}\partial_w\phi\partial_w\overline{\phi}{:} - \frac{1}{6}\{w,z\} \tag{4.8}$$

where $\{w,z\}$ — the Schwarzian derivative — is

$$\{w,z\} = \frac{w'''(z)}{w'(z)} - \frac{3}{2}\left(\frac{w''(z)}{w'(z)}\right)^2 \tag{4.9}$$

We are interested in the transformation properties of the currents $j^{(s)}(u)$ when $w(z)$ is

$$w(z) = -e^{-\kappa z} \tag{4.10}$$

In [11] we obtained

$$j^{(s)}_{z\ldots z}(z) \to \left(\frac{1}{\kappa w}\right)^s \left(j^{(s)}_{z\ldots z} + \langle X\rangle_s\right) \tag{4.11}$$

where

$$\langle X_s\rangle = (-)^{s-1}(-i)^{s-2}\kappa^s\frac{B_s}{s} \tag{4.12}$$

Eq. (4.11) has to be compared with eq. (2.16). This is a higher order Schwarzian derivative evaluated at $w(z) = -e^{-\kappa z}$. It plays a role analogous to the r.h.s. of (2.17). Below we will compare it with the radiation moments in the r.h.s. of (4.2).

4.2 Higher spin covariant currents

Let us now return to the light-cone notation. We identify $j^{(2)}_{uu}(u)$ up to a constant with the holomorphic energy momentum tensor

$$j^{(2)}_{uu}(u) = -2\pi\, T^{(hol)}_{uu} \tag{4.13}$$

Similarly we identify $j^{(s)}_{u\ldots u}$, with s lower indices, with an s-th order holomorphic tensor. They can be naturally thought of as the only non-vanishing components of a two-dimensional completely symmetric current. In analogy with the energy-momentum tensor, we expect that there exist a conformally covariant version $J^{(s)}_{u\ldots u}$ of $j^{(s)}_{u\ldots u}$. The latter must be the intrinsic component of a two-dimensional completely symmetric traceless current $J^{(s)}_{\mu_1\ldots\mu_s}$, whose only other classically non-vanishing component is $J^{(s)}_{v\ldots v}$. We identify them with the currents (4.28)

The previous holomorphic currents refer to a background with trivial (Euclidean) metric. In order to find a covariant expression of them we have to be able to incorporate the information of a non-trivial metric. This was done in [11] following [7]. According to the recipe explained there, the covariant counterpart of $j^{(s)}_{u\ldots u}$ should be constructed using currents

$$J^{(n,m)}_{u\ldots u} = e^{(n+m)\varphi(u)} \lim_{\epsilon\to 0}\left\{ e^{-n\varphi(u_1)-m\varphi(u_2)}\nabla^n_{u_1}\phi\nabla^m_{u_2}\overline{\phi} - \frac{c_{n,m}\hbar}{\epsilon^{n+m}}\right\} \tag{4.14}$$

where $c_{m,n} = (-)^m(n+m-1)!$ are numerical constants determined in such a way that all singularities are canceled in the final expression for $J^{(n,m)}_{u\ldots u}$. Therefore (4.14) defines the normal ordered current

$$J^{(n,m)}_{u\ldots u} = :\nabla^n_u\phi\nabla^m_u\overline{\phi}: \tag{4.15}$$

After some algebra one gets

$$\begin{aligned} J^{(2)}_{uu} &= j^{(2)}_{uu} - \frac{\hbar}{6}\mathcal{T} \\ J^{(3)}_{uuu} &= j^{(3)}_{uuu} \\ J^{(4)}_{uuuu} &= j^{(4)}_{uuuu} + \frac{\hbar}{30}\mathcal{T}^2 + \frac{2}{5}\mathcal{T}J^{(2)}_{uu} \\ J^{(5)}_{uuuuu} &= j^{(5)}_{uuuuu} + \frac{10}{7}\mathcal{T}J^{(3)}_{uuu} \end{aligned} \tag{4.16}$$

and

$$\begin{aligned} J^{(6)}_{uuuuuu} = \Big(& -\frac{2\hbar}{63}\mathcal{T}^3 + \frac{5\hbar}{504}(\partial_u\mathcal{T})^2 - \frac{\hbar}{126}\mathcal{T}\partial_u^2\mathcal{T} \\ & -\frac{2}{3}\mathcal{T}^2 J^{(2)}_{uu} - \frac{1}{21}\mathcal{T}\nabla_u^2 J^{(2)}_{uu} - \frac{1}{21}\left(\partial_u^2\mathcal{T}\right)J^{(2)}_{uu} + \frac{5}{42}\left(\partial_u\mathcal{T}\right)\nabla_u J^{(2)}_{uu} \\ & -\frac{5}{21}\Gamma\mathcal{T}\nabla_u J^{(2)}_{uu} - \frac{5}{21}\Gamma^2\mathcal{T}J^{(2)}_{uu} + \frac{5}{21}\Gamma\left(\partial_u\mathcal{T}\right)J^{(2)}_{uu}\Big) - \frac{5}{24}\mathcal{T}J^{(4)}_{uuuu} + j^{(6)}_{uuuuuu} \end{aligned} \tag{4.17}$$

where

$$\mathcal{T} = \partial_u^2\varphi - \frac{1}{2}\left(\partial_u\varphi\right)^2 \tag{4.18}$$

These equations are the analogs of (3.9).

The covariant divergences of these currents are

$$g^{uv}\nabla_v J^{(2)}_{uu} = \frac{\hbar}{12}(\nabla_u R) \tag{4.19}$$

$$g^{uv}\nabla_v J^{(3)}_{uuu} = 0 \tag{4.20}$$

$$g^{uv}\nabla_v J^{(4)}_{uuuu} + \frac{1}{5}q^2 (\nabla_u R) J^{(2)}_{uu} = 0 \tag{4.21}$$

$$g^{uv}\nabla_v J^{(5)}_{uuuuu} + \frac{5}{7}(\nabla_u R) J^{(3)}_{uuu} = 0 \tag{4.22}$$

and, for $s = 6$

$$g^{uv}\nabla_v J^{(6)}_{uuuuuu} + \left(\frac{5}{84}(\nabla_u^2 R)\nabla_u J^{(2)}_{uu} - \frac{1}{42}(\nabla_u R)\nabla_u^2 J^{(2)}_{uu} - \frac{1}{42}(\nabla_u^3 R) J^{(2)}_{uu}\right) + \frac{5}{3}(\nabla_u R) J^{(4)}_{uuuu} = 0 \tag{4.23}$$

Eq. (4.19) is to be compared with (3.7) while the remaining ones are the relevant higher spin analogs.

The above equations mean that all the higher spin equations are covariantly conserved. In the r.h.s. of (4.20)–(4.23), unlike (4.19), there does not appear any terms proportional to $\hbar$. Any such term must be interpreted as the consequence of a trace anomaly (and possibly a diff anomaly) as has been argued by [9]. In other words if there is a term proportional to $\hbar$ in $g^{uv}\nabla_v J_{uu\ldots u}$ this must be understood as related to the second term in the covariant divergence $\nabla^\mu J_{\mu u\ldots u} = g^{uv}\nabla_v J_{uu\ldots u} + g^{uv}\nabla_u J_{vu\ldots u}$. Such a term tells us that $J_{vu\ldots u}$, which classically vanishes, takes on a nonzero value at one loop, revealing the existence of a trace anomaly. This is precisely what happens for the covariant second order current (energy-momentum tensor) $J^{(2)}_{\mu\nu}$ (4.19): the trace is $\mathrm{Tr}(J^{(2)}) = 2g^{vu}J^{(2)}{}_{vu}$. Thus, (4.19) reproduces the well known trace anomaly $\mathrm{Tr}(J^{(2)}) = -\frac{c\hbar}{12}R$, where in our case $c_R = 2$.[4]

However for the other equations, we see that the terms that carry explicit factors of $\hbar$ cancel out in eqs. (4.20)–(4.23). This implies the absence of $\hbar$ terms in the trace, and consequently the absence of any trace anomaly as well as of any diffeomorphism anomaly.

In [11] it was shown that, as far as trace anomalies are concerned, this result is to be expected, since via a cohomological analysis it can be seen that no true trace anomaly can exist in higher spin currents.

Of course we could repeat the same construction for antiholomorphic currents and find the corresponding covariant ones. We would find perfectly symmetric results with respect to the ones above.

4.3 Higher moments of the Hawking radiation

Now let us apply to the just introduced higher spin currents an argument similar to the one in section 2 for the energy-momentum tensor, using the previous results from the W_∞

[4]We relate $j^{(2)}_{uu}$ to the energy momentum tensor via the factor of 2π and the minus sign. This is because in the Euclidean we want to conform to the conventions and results of [56], where properly normalized currents satisfy a W_∞ algebra. This holds for higher order currents too: for physical applications their W_∞ representatives must all be divided by -2π.

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algebra. Introducing the Kruskal coordinate $U = -e^{-\kappa u}$ and requiring regularity at the horizon we find that, at the horizon, the value of $j^{(s)}_{u\dots u}$ is given by $\langle X_s\rangle$ in eq. (4.12). Next $j^{(s)}_{u\dots u}(u)$ is constant in t and r (the same is of course true for $j^{(s)}_{v\dots v}$). Therefore, if we identify $j^{(s)}_{u\dots u}(u)$ with $j^{(s)}_{z\dots z}(z)$ via Wick rotation, $\langle X_s\rangle$ corresponds to its value at $r=\infty$. Since $j^{(s)}_{u\dots u}(u)$ and $J^{(s)}_{u\dots u}(u)$ asymptotically coincide, the asymptotic flux of these currents is

$$-\frac{1}{2\pi}\,\langle J^{(s)^r}{}_{t\dots t}\rangle = -\frac{1}{2\pi}\,\langle J^{(s)}_{u\dots u}\rangle + \frac{1}{2\pi}\,\langle J^{(s)}_{v\dots v}\rangle = -\frac{1}{2\pi}\,\langle X_s\rangle = \frac{i^{s-2}}{2\pi s}\kappa^s B_s \qquad (4.24)$$

For the global -2π factor, see the previous footnote.

The r.h.s. vanishes for odd s (except $s=1$ which is not excited in our case) and coincides with the thermal flux moments (4.2) for even s.

4.4 A qualitative motivation for higher spin currents

We would like to spend a few words concerning the origin of higher spin currents, even though what follows is very qualitative and is in fact not needed in the economy of the paper.

Let us suppose we know the energy momentum tensor of a fundamental theory which faithfully reproduces the full spectrum of the Hawking radiation and expand it around our background metric. To guess what may occur think of a quantum energy-momentum tensor represented in the Sugawara form in a flat background: $T_{\mu\nu} = (: J_\mu J_\nu : -\text{trace})$, where, for instance, $J_\mu = \partial_\mu\phi$ in the simplest case. We can view it as an expression point-split by a small but finite amount $2y$

$$T_{\mu\nu}(x) = \lim_{y\to 0} : \partial_\mu\phi(x-y)\partial_\nu\phi(x+y) - \text{trace} : \qquad (4.25)$$

The finite point splitting is meant to account for a nonlocal interaction that synthesizes the interactions of the underlying model (see the related considerations in [7]). Let us expand in Taylor series

$$\begin{aligned} &: \partial_\mu\phi(x-y)\partial_\nu\phi(x+y) : \\ &\quad = \sum_{i=0}\sum_{j=0}\frac{(-1)^i}{i!j!} : y^{\mu_1}\dots y^{\mu_i}\partial_\mu\partial_{\mu_1}\dots\partial_{\mu_i}\phi(x) y^{\nu_1}\dots y^{\nu_j}\partial_\nu\partial_{\nu_1}\dots\partial_{\nu_j}\phi(x) : \end{aligned} \qquad (4.26)$$

This expansion is appropriate for a two-dimensional flat space-time, but we will need to consider point splitting in a curved space-time. Therefore in (4.26) the derivative will be replaced by covariant derivative and the products $y^{\mu_1}\dots y^{\mu_i}y^{\nu_1}\dots y^{\nu_j}$ by complicated expressions of the background. We represent all this by effective background tensor fields $B^{(s)}_{\mu_1\dots\mu_s}$. When inserted back in (4.25), the quantum expression will give rise to an expansion of the energy-momentum tensor in terms of higher spin currents coupled to such fields.

In a previous subsection we have constructed higher spin currents from a W_∞ algebra using a chiral coordinate z, which we understand as the local holomorphic coordinate over a Riemann surface Σ. A W_∞ algebra is generated on a local patch not only by diffeomorphisms, but by more general coordinate transformations, the symplectomorphisms, which involve also the cotangent bundle of Σ, see [61] and, for an explicit construction, [62]. In particular in [62] it is shown that from general transformations of the type

$$\delta C^{(r)}(z,\bar{z}) = \sum_{s=1}^{r} s\, C^{(s)}(z,\bar{z})\,\partial_z C^{(r-s-2)}(z,\bar{z}) \tag{4.27}$$

where $C^{(r)}$ are 'ghost' tensors of order r, the following algebra follows for an infinite set of generators $T^{(r)}(z,\bar{z})$,

$$\Big[T^{(r)}(z,\bar{z}), T^{(s)}(z',\bar{z}')\Big] = (r-1)\partial^{z'}\delta(z'-z)T^{r+s-2}(z,\bar{z}) - (s-1)\partial_z\delta(z-z')T^{r+s-2}(z',\bar{z}')$$

This is the classical version of the W_∞ algebra (for a quantum version, see for instance [56]). It is possible to recognize in (4.27) the transformations (5.4) and (5.5) below.

Now the above expression (4.25) exhibits a dependence both on x^μ and on $y^\mu \equiv dx^\mu$. We can think of y^μ as local coordinates on the cotangent bundle of Σ and their transformations can be conceived of as W_∞ transformations. Therefore, even though the details fully depend on the fundamental theory and remain implicit, the appearance of higher spin currents and their W_∞ algebra structure is not so surprising.

Each of these higher spin currents carries to infinity its own piece of information about the Hawking radiation. Just in the same way as in the action the metric is a source for the energy-momentum tensor, these new (covariant) currents will have in the effective action suitable sources, with the appropriate indices and symmetries. In [11] they were represented by asymptotically trivial background fields $B^{(s)}_{\mu_1\ldots\mu_s}$ (in [61] they were called 'cometric functions'). So we have

$$J^{(s)}_{\mu_1\ldots\mu_s} = \frac{1}{\sqrt{g}}\frac{\delta}{\delta B^{(s)\mu_1\ldots\mu_s}} S \tag{4.28}$$

In particular $B^{(2)}_{\mu\nu} = g_{\mu\nu}/2$. We assume that all $J^{(s)}_{\mu_1\ldots\mu_s}$ are maximally symmetric and classically traceless.

5. Diffeomorphism anomalies for higher spin currents

In subsection 4.2 we saw that it is consistent to require that higher spin currents are covariantly conserved. This leads for such higher tensor currents to the absence both of trace and diffeomeorphisms anomalies. The trace and covariant divergence of the currents were determined with a particular construction based on currents made out of a bosonic scalar field. Therefore it is important to find an independent confirmation of such results.

As for the trace anomalies it was shown in [11] that this is no accident: the trace of the fourth order current does not admit true anomalies (there may appear anomalous terms, but they correspond to trivial cocycles and can be canceled by suitable counterterms in the effective action). This result is seemingly valid for all the higher spin currents, because a thumb rule suggests that true anomalies appear only when the cocycle engineering dimension (in our case the total number of derivatives) is related in a precise way to the space-time dimension.

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As for the diffeomorphism anomalies, on the basis of the previous construction there is no evidence of them either. But in [7] some diff anomalies appeared in the covariant divergence of higher spin (bi-spinorial) currents. It is therefore important to verify that this is not in contrast with our results above. This means that we have to prove that such anomalies are trivial. Eqs. (4.19) through (4.23), are covariant conservation equations (as it is apparent in eq. (4.19)). Therefore, if anomalies ever appear in such conservation equations, they appear in covariant form. Existence or non-existence of covariant anomalies is not easy to analyze in general, while general results can be obtained for consistent anomalies. Since absence of consistent anomalies implies absence of the corresponding covariant ones, we will try to show that, for the conservation laws we are interested in, there are no consistent anomalies (except the well-known one corresponding to (4.19)). It should be remarked that this problem is interesting in itself, even independently of the application considered in this paper, and, to our best knowledge, has not been studied so far.

In the sequel we will give for the fourth order current a proof of absence of diff anomalies analogous to the one that was presented in [11] for trace anomalies and, under reasonable assumptions, we will extend the proof to currents of any order. This will lend support to our previous claims, beyond the explicit construction of the previous section.

5.1 The consistency method for diff anomalies

The conservation of the energy-momentum tensor corresponds, as is well-known, to the symmetry of the theory under the diffeomorphism transformations:

$$\delta_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \tag{5.1}$$

where $\xi_\mu = g_{\mu\nu}\xi^\nu$, and ξ^μ represent infinitesimal general coordinate transformations $x^\mu \to x^\mu + \xi^\mu$. The background fields transform in a covariant way under these transformation

$$\delta_\xi B^{(s)}_{\mu_1\ldots\mu_s} = \xi^\lambda \partial_\lambda B^{(s)}_{\mu_1\ldots\mu_s} + \partial_{\mu_1}\xi^\lambda B^{(s)}_{\lambda\ldots\mu_s} + \cdots + \partial_{\mu_s}\xi^\lambda B^{(s)}_{\mu_1\ldots\lambda} \tag{5.2}$$

Similarly the conservations of higher spin currents correspond to the symmetry under higher tensorial transformation. In particular the conservation of $J^{(4)}$ is due to invariance under

$$\delta_\tau B^{(4)}_{\mu_1\mu_2\mu_3\mu_4} = \nabla_{\mu_1}\tau_{\mu_2\mu_3\mu_4} + \text{cycl.} \tag{5.3}$$

where τ is a completely symmetric traceless tensor and cycl denotes cyclic permutations of the indices. The reason for tracelessness will be given later.

To find the (consistent) anomalies of the energy-momentum tensor and higher spin currents with respect to the symmetry induced by the above transformations, we will analyze the solutions of the relevant Wess-Zumino consistency conditions. An equivalent (and simpler) way is to transform the problem into a cohomological one. The trick is well-known. We promote the transformation parameters to anticommuting ghost fields and endow them with a suitable transformation law. This gives rise to a nilpotent operator acting on the local functionals of the fields and their derivatives. Local functionals (cochains) and nilpotent operator (coboundary) define a differential complex. Anomalies correspond to non-trivial cocycles.

For ξ this leads to

$$\delta_\xi \xi^\mu = \xi^\lambda \partial_\lambda \xi^\mu \tag{5.4}$$

beside

$$\delta_\xi \tau_{\mu\nu\rho} = \xi^\lambda \partial_\lambda \tau_{\mu\nu\rho} + \partial_\mu \xi^\lambda \tau_{\lambda\nu\rho} + \partial_\nu \xi^\lambda \tau_{\mu\lambda\rho} + \partial_\rho \xi^\lambda \tau_{\mu\nu\lambda} \tag{5.5}$$

It is then easy to show that $\delta_\xi^2 = 0$.

In a similar way, beside $\delta_\tau g_{\mu\nu} = 0$, we set

$$\delta_\tau \tau_{\mu\nu\lambda} = 0 \tag{5.6}$$

that is, we assume that τ is an Abelian parameter. This is not obvious a priori and requires a specific justification. We do it in appendix A. Now it is elementary to prove that, as a consequence of the anticommutativity of τ we have $\delta_\tau^2 = 0$. More generally, since τ is assumed to anticommute with ξ, we have

$$\delta_\xi^2 = 0, \qquad \delta_\tau^2 = 0, \qquad \delta_\xi \delta_\tau + \delta_\tau \delta_\xi = 0 \tag{5.7}$$

In the following we will denote by δ_τ, δ_ξ also the corresponding functional operators. It follows from (5.7) that the operator $\delta_{tr} = \delta_\xi + \delta_\tau$ is nilpotent. It is clear that δ_{tr} is not the total functional operator of our system, but rather a truncated one, since we are disregarding higher tensorial gauge transformations.[5] Such a truncation is justified by the fact that our differential system is graded. This can be seen as follows.

Let us recall first the canonical dimensions of the various fields involved. $g_{\mu\nu}$ has dimensions 0; ξ has dimension (in mass) -1, while $B^{(4)}$ and τ have dimensions -2 and -3, respectively. Now let us consider the nilpotent total differential operator $\delta_{\rm tot} = \delta_\xi + \delta_\tau + \cdots$. Then (integrated) anomalies are defined by

$$\delta_{\rm tot}\, \Gamma^{(1)} = \hbar \Delta, \qquad \delta_{\rm tot} \Delta = 0 \tag{5.8}$$

where $\Gamma^{(1)}$ is the one-loop quantum action. Δ, which is the integral of a local functional in the fields and their derivatives, splits naturally into $\Delta_\xi + \Delta_\tau + \cdots$. In turn each addend splits into a sum of terms according to the degree of their integrand. The degree is defined by the number of derivative of the integrand minus 1. Therefore we have for instance

$$\Delta_\xi = \Delta_\xi^{(2)} + \Delta_\xi^{(4)} + \Delta_\xi^{(6)} + \cdots, \qquad \Delta_\tau = \Delta_\tau^{(4)} + \Delta_\tau^{(6)} + \cdots$$

As a consequence $\delta_{\rm tot}\Delta = 0$ splits into

$$\delta_\xi \Delta_\xi^{(2)} = 0 \tag{5.9}$$

$$\delta_\xi \Delta_\xi^{(4)} = 0 \tag{5.10}$$

$$\delta_\xi \Delta_\xi^{(6)} = 0, \quad \ldots \tag{5.11}$$

[5] To be precise we are concentrating on eqs. (4.19), (4.21) and disregarding (4.23). The conservation laws (4.20) and (4.22) do not admit anomalies in the present context.

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and

$$\delta_\tau \Delta_\tau^{(4)} = 0 \tag{5.12}$$
$$\delta_\tau \Delta_\tau^{(6)} = 0, \quad \dots \tag{5.13}$$

with the cross conditions[6]

$$\delta_\tau \Delta_\xi^{(4)} + \delta_\xi \Delta_\tau^{(4)} = 0 \tag{5.14}$$
$$\delta_\tau \Delta_\xi^{(6)} + \delta_\xi \Delta_\tau^{(6)} = 0, \quad \dots \tag{5.15}$$

Therefore, fortunately, our complex splits into subcomplexes and, for example it makes sense to truncate it at level 4, i.e. to eqs. (5.9), (5.10), (5.12) and (5.14), since these conditions are not affected by the higher order equations in the complex.

5.2 The search for δ_τ anomalies

Let us explain the strategy to prove the absence of anomalies for fourth order currents. The first step is to solve eqs. (5.9), (5.10) in general. We will show that, while (5.9) admits a nontrivial solution (the 2d diff anomaly), (5.10) does not admit any nontrivial solution. This will be done in appendix B: the proof is based on an argument used for 4d anomalies in [65] and adapted to the present context. What we prove precisely is that any solution to eq. (5.10) is trivial, that is there exist a local functional $C^{(4)}$ of the background fields such that if $\Delta_\xi^{(4)}$ is a solution to (5.10), then $\Delta_\xi^{(4)} = \delta_\xi C^{(4)}$. Therefore we can rewrite (5.14) as

$$\delta_\xi(\Delta_\tau^{(4)} - \delta_\tau C^{(4)}) = 0 \tag{5.16}$$

This amounts to saying that any cocycle of δ_τ (i.e. any solution to (5.12)) can be written in a diff-covariant form. This is a piece of very useful information because it strongly limits the forms of the cochains we have to analyze in order to find the solutions to (5.12).

What remains for us to do is very simple. Let us start with an example. We write a first set of chains

$$\Delta_\tau = \int d^2x \sqrt{-g} \sum_{i=1}^{3} a_i\, I_i^\tau \tag{5.17}$$

where

$$I_1^\tau = \tau^{\mu\nu\lambda}\, \nabla_\mu \nabla_\nu \nabla_\lambda R, \qquad I_2^\tau = \tau^{\mu\lambda}{}_\lambda\, \Box \nabla_\mu R, \qquad I_3^\tau = \tau^{\mu\lambda}{}_\lambda\, \nabla_\mu R^2$$

where we have ignored tracelessness of τ. All these cochains are, trivially, cocycles of δ_τ and they are the only ones one can construct of this type.[7]

---

[6] The action of δ_τ on $\Delta_\xi^{(2)}$ is trivial.

[7] In 2 dimensions we have

$$\begin{aligned} R_{\mu\nu\lambda\rho} &= \frac{1}{2} R \,(g_{\mu\lambda}\, g_{\nu\rho} - g_{\mu\rho}\, g_{\nu\lambda}) \\ R_{\mu\nu} &= \frac{1}{2} g_{\mu\nu}\, R \end{aligned} \tag{5.18}$$

Next we have to find out whether these cocycles are trivial or not. The only possible counterterms are also 3.

$$C = \int d^2x \sqrt{-g} \sum_{j=1}^{3} c_j J_j \tag{5.19}$$

where

$$J_1 = B^{\mu\nu\lambda}{}_\lambda \nabla_\mu \nabla_\nu R, \qquad J_2 = B^{\mu\lambda}{}_{\mu\lambda} \nabla^\nu \nabla_\nu R, \qquad J_3 = B^{\mu\lambda}{}_{\mu\lambda} R^2$$

Applying δ_τ to (5.19) we get

$$\delta_\tau C = \int d^2x \sqrt{-g} \sum_{i,j=1}^{3} c_i M_{ij} I_j^\tau \tag{5.20}$$

where M_{ij} is the matrix

$$M_{ij} = - \begin{pmatrix} 2 & 2 & 0 \\ 0 & 4 & -1 \\ 0 & 0 & 4 \end{pmatrix} \tag{5.21}$$

Since the determinant of this matrix is nonvanishing we can always find c_i such that (5.20) reproduce (5.17) for any choice of the parameters a_i. Therefore all the cocycles (5.17) are trivial.

This is not enough since the cocycles (5.17) are not of the most general form. We expect a true diff anomaly to contain the $\epsilon_{\mu\nu}$ tensor (see section 3). There are three cochains of such a form

$$\Delta_\tau = \int d^2x \sqrt{-g} \sum_{i=1}^{3} b_i K_i^\tau \tag{5.22}$$

where

$$K_1^\tau = \tau^{\mu\nu\lambda} \epsilon_{\mu\alpha} \nabla^\alpha \nabla_\nu \nabla_\lambda R, \qquad K_2^\tau = \tau^{\mu\lambda}{}_\lambda \epsilon_{\mu\alpha} \Box \nabla^\alpha R, \qquad K_3^\tau = \tau^{\mu\lambda}{}_\lambda \epsilon_{\mu\alpha} \nabla^\alpha R^2 \tag{5.23}$$

They are, trivially, cocycles.

On the other hand now there is only one possible counterterm

$$C = \int d^2x \sqrt{-g} B^{\mu\nu\lambda}{}_\lambda \epsilon_{\nu\alpha} \nabla^\alpha \nabla_\mu R \tag{5.24}$$

It it easy to see that

$$\delta_\tau C = - \int d^2x \sqrt{-g} \left(2\tau^{\mu\lambda\rho} \epsilon_{\rho\alpha} \nabla^\alpha \nabla_\lambda \nabla_\mu R + \tau^{\mu\lambda}{}_\lambda \epsilon_{\mu\alpha} \Box \nabla^\alpha R + 2\tau^{\mu\lambda}{}_\lambda \epsilon_{\mu\alpha} R \nabla^\alpha R \right) \tag{5.25}$$

Therefore this counterterm is not enough to cancel the three previous independent cocycles. Here come tracelessness of τ. This property is necessary because it is easy to realize that the last two terms in (5.23), which are proportional to $\tau^{\mu\lambda}{}_\lambda$, would appear in conservation laws in which also the components $J^{(4)}_{uuvv}$ are 'excited'. This would bring us outside our system. To avoid this we have to impose that τ is traceless. This being so, only K_1^τ survives among the cocycles, and only the first term survives in the r.h.s. of (5.25). The latter precisely cancels the only possible nontrivial cocycle.

To conclude, there are no non-trivial consistent anomalies in the divergence of the fourth order current.

It is not hard to extend the above argument to sixth and higher order currents, provided we assume that all the chains can be written in a covariant form. This corresponds to assuming that there are no non-trivial solutions to eq. (5.11) and the analogous higher equations. Proving this result requires a refinement of the techniques used in appendix B, and we will not do it here. However it is very reasonable to assume it.

Let us prove the following claim: all solutions to the equation

$$\delta_\omega \Delta^{(2n)}_\omega = 0$$

where $\omega^{\mu_1 \cdots \mu_{2n-1}}$ is a totally symmetric, traceless ghost parameter (the generalization of $\tau_{\mu\nu\lambda}$), are trivial, i. e. there exists a local functional $C^{(2n)}$ of the background fields, such that

$$\Delta^{(2n)}_\omega = \delta_\omega C^{(2n)}.$$

We will show this under the assumption that all chains δ_ω acts upon can be written in a diff-covariant form. Therefore we start by writing the most general cocycles as

$$\Delta_\omega = \int d^2x \sqrt{-g}\,(aI^\omega + bK^\omega) \tag{5.26}$$

where a and b are constants and I^ω and K^ω are the *only* possible terms (see appendix B) we can construct in $D = 2$, taking into account the tracelessness of ω. Their explicit form is:

$$I_\omega = \omega^{\mu_1 \cdots \mu_{2n-1}} \nabla_{\mu_1} \ldots \nabla_{\mu_{2n-1}} R \tag{5.27}$$

and

$$K_\omega = \omega^{\mu_1 \cdots \mu_{2n-1}}\, \epsilon_{\mu_1 \alpha}\, \nabla^\alpha \nabla_{\mu_2} \ldots \nabla_{\mu_{2n-1}} R \tag{5.28}$$

Now we claim that the corresponding counterterm is the following:

$$C = -\frac{1}{2} \int d^2x \sqrt{-g}\,(aJ^\omega + bL^\omega) \tag{5.29}$$

where

$$J_\omega = B^{\mu_1 \cdots \mu_{2n-2}\sigma}{}_\sigma\, \nabla_{\mu_1} \ldots \nabla_{\mu_{2n-2}} R \tag{5.30}$$

and

$$L_\omega = B^{\mu_1 \cdots \mu_{2n-2}\sigma}{}_\sigma\, \epsilon_{\mu_1 \alpha} \nabla^\alpha \nabla_{\mu_2} \ldots \nabla_{\mu_{2n-2}} R \tag{5.31}$$

and B is the corresponding background field. Using the formulas

$$\begin{aligned}\delta_\omega g_{\mu\nu} &= 0\\ \delta_\omega B^{\mu_1\dots\mu_{2n}} &= \nabla^{\mu_1}\omega^{\mu_2\dots\mu_{2n}} + \text{cycl.}\end{aligned}$$

and again the fact that ω is traceless, we get, after integration by parts,

$$\begin{aligned}\delta_\omega C = \int d^2x\sqrt{-g}\,\big(a\,\omega^{\mu_1\dots\mu_{2n-2}\sigma}\nabla_\sigma\nabla_{\mu_1}\dots\nabla_{\mu_{2n-2}}R\, +\\ +\, b\,\omega^{\mu_1\dots\mu_{2n-2}\sigma}\epsilon_{\mu_1\alpha}\nabla_\sigma\nabla^\alpha\nabla_{\mu_2}\dots\nabla_{\mu_{2n-2}}R\big)\end{aligned}$$

The second term under the integral has to be rearranged by reversing the order of the first two covariant derivatives ($\nabla_\sigma\nabla^\alpha$). Using the formulas in the previous footnote, we have

$$\begin{aligned}\omega^{\mu_1\dots\mu_{2n-2}\sigma}\epsilon_{\mu_1\alpha}\nabla_\sigma\nabla^\alpha\nabla_{\mu_2}\dots\nabla_{\mu_{2n-2}}R = \omega^{\mu_1\dots\mu_{2n-2}\sigma}\epsilon_{\mu_1\alpha}\nabla^\alpha\nabla_\sigma\nabla_{\mu_2}\dots\nabla_{\mu_{2n-2}}R\\ +\omega^{\mu_1\dots\mu_{2n-2}\sigma}\epsilon_{\mu_1\alpha}R_\sigma{}^\alpha{}_{\mu_2}{}^\lambda\nabla_\lambda\nabla_{\mu_3}\dots\nabla_{\mu_{2n-2}}R + \dots\end{aligned}$$

So, a typical additional term has a form

$$\frac{R}{2}\,\omega^{\mu_1\dots\mu_{2n-2}\sigma}\,\epsilon_{\mu_1\alpha}\,(g_{\sigma\mu_i}\,g^{\alpha\lambda} - g_\sigma^\lambda\,g_{\mu_i}^\alpha)\nabla_{\mu_2}\dots\nabla_{\mu_{i-1}}\nabla_\lambda\nabla_{\mu_{i+1}}\dots\nabla_{\mu_{2n-2}}R$$

The first part of this term vanishes because of the tracelessness of ω and the second because it leads to contraction of antisymmetric ϵ tensor and symmetric indices in ω. Therefore, we have proven that

$$\delta_\omega C = \int d^2x\sqrt{-g}\,(aI^\omega + bK^\omega) \tag{5.32}$$

This means that, allowing for the above assumption, there are no non-trivial anomalies in any higher spin currents. Therefore a properly chosen regularization should not produce any covariant anomaly either. This is reflected in our eqs. (4.21) and (4.23), which express the covariant conservation of the fourth and sixth order currents. The additional terms in the l.h.s. (which are not present in the consistent version of the conservation law) are needed in order to guarantee covariance of the divergence in the presence of the non-trivial gravitational background (see appendix A).

6. Conclusions

In this paper we have shown that the two methods of calculating the integrated flux of Hawking radiation on a static symmetric black hole, the method that makes use of the trace anomaly and the one based on the diffeomorphism anomaly, are strictly related. The two methods actually boil down to the same basic elements. We have also pointed out the basic role of the integrated conservation equations (3.9) and (3.10).

In order to describe the higher moments of the Hawking radiation spectrum, we have introduced higher spin currents. They have been constructed starting from a W_∞ algebra on the complex plane and subsequently lifted to the curved space-time corresponding to the

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black hole background metric. They were shown in [11] to describe the higher moments of the black hole emission. We passed then to analyze the presence of anomalies in the traces and covariant divergences of these higher tensorial currents. The above mentioned explicit construction reveals none. Therefore we went on to analyze the possible existence of higher order trace and diff anomalies, relying on consistency methods (Wess-Zumino consistency conditions). In [11] it was shown that no trace anomaly exists for the fourth order current. In this paper we have analyzed the most challenging problem of diff anomalies. The result is still negative: no non-trivial anomalies exist.

The extension of the anomaly analysis to still higher orders is very challenging, but we believe that we have gathered enough evidence that higher spin currents cannot have anomalies, only the energy-momentum tensor can. This corresponds to a prejudice according to which anomalies exists only when a precise relation exists between number of derivatives and space-time dimensions. It is also suggested by the presence of a unique central charge in the underlying W_∞ algebra.

On the other hand anomalies are not necessary to describe higher moments of the Hawking radiation. Rather, the properties of the W_∞ algebra offer a convincing explanation for them.[8] We therefore conclude our analysis with the claim that the universal character of the Hawking fluxes has its basis in a W_∞ algebra underlying the matter model for radiation.

Acknowledgments

We would like to thank R.Banerjee, S.Iso and H.Umetsu for their kind and useful messages and P.Dominis Prester for useful discussions. M.C. would like to thank SISSA for hospitality and Central European Initiative (CEI) and The National Foundation for Science, Higher Education and Technological Development of the Republic of Croatia (NZZ) for financial support. I.S.,M.C and S.P. would like to acknowledge support by the Croatian Ministry of Science, Education and Sport under the contract no.119-0982930-1016

A. The τ transformations

In this appendix we would like to discuss the nature of the τ transformations and argue that they are abelian. Let us start from the second term in the l.h.s. of (4.21), a term which does not appear in the consistent version of the conservation law. The l.h.s. of (4.21) is formally generated by the variation of the action with respect to τ given by (5.3) *and* by

$$\delta_\tau g_{\mu\nu} = a\nabla^\lambda R\,\tau_{\mu\nu\lambda} \tag{A.1}$$

where $a = -\frac{1}{5}$.

The presence of this nontrivial transformation of the metric under τ changes completely the rules laid down in section 5.1. Therefore we must ask ourselves whether (A.1) is a true

[8] After this article was posted on the archive, S.Iso and H.Umetsu pointed out to us that our result has an additional valence: higher spin anomalies would give rise to a new kind of 'hairs' corresponding to higher spin central charges; therefore our proof of the absence of such anomalies shows the agreement of the Hawking radiation analysis with the no-hair theorem.

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symmetry transformation or a simple functional variation of the fields necessary in order to derive a covariant conservation law. We will argue here that that the second alternative is the correct one. Therefore (A.1) is not a symmetry operation and the rules of section 5.1 are correct, in particular the τ transformation rules are abelian. But, let us explain this in stages.

The way eq. (4.21) was obtained does not allow us to conclude whether it represents a consistent or covariant conservation law. However the functional variation (A.1) contains a non-universal factor a (a varies according to the regularization and the model) which should tell us that the latter cannot be a symmetry operation. However, short of a conclusive argument, we can try to embed (A.1) as well as (5.3) in a new set of transformations, where possibly $\delta_\tau \tau \neq 0$, and see whether we can implement a new group theoretical transformation. This is guaranteed if the corresponding functional operator δ_τ turns out to be nilpotent. However, as we shall see, this is not the case.

Let us consider a general form for variation $\delta_\tau g_{\mu\nu}$

$$\delta_\tau g_{\mu\nu} = \sum_{i=1}^{12} a_i I^i_{(\mu\nu)} \tag{A.2}$$

where

$$\begin{aligned}
I^1_{\mu\nu} &= \nabla_\mu \nabla_\nu \nabla^\alpha \tau_\alpha{}^\beta{}_\beta \\
I^2_{\mu\nu} &= \nabla^\alpha \nabla_\alpha \nabla_\mu \tau_\nu{}^\beta{}_\beta \\
I^3_{\mu\nu} &= \nabla^\alpha \nabla^\beta \nabla_\mu \tau_{\nu\alpha\beta} \\
I^4_{\mu\nu} &= \nabla^\alpha \nabla_\alpha \nabla^\beta \tau_{\mu\nu\beta} \\
I^5_{\mu\nu} &= \nabla^\alpha \nabla^\beta \nabla^\gamma \tau_{\alpha\beta\gamma} g_{\mu\nu} \\
I^6_{\mu\nu} &= \nabla^\alpha \nabla_\alpha \nabla^\beta \tau_\beta{}^\gamma{}_\gamma g_{\mu\nu} \\
I^7_{\mu\nu} &= R \nabla^\alpha \tau_{\mu\nu\alpha} \\
I^8_{\mu\nu} &= R \nabla_\mu \tau_\nu{}^\alpha{}_\alpha \\
I^9_{\mu\nu} &= R \nabla^\alpha \tau_\alpha{}^\beta{}_\beta g_{\mu\nu} \\
I^{10}_{\mu\nu} &= \nabla^\alpha R \tau_{\mu\nu\alpha} \\
I^{11}_{\mu\nu} &= \nabla_\mu R \tau_\nu{}^\alpha{}_\alpha \\
I^{12}_{\mu\nu} &= \nabla^\alpha R \tau_\alpha{}^\beta{}_\beta g_{\mu\nu}
\end{aligned} \tag{A.3}$$

We look at the possible constraints on the coefficients a_i in (A.2) that come from nilpotence of δ_τ. Acting with δ_τ on (5.3) we obtain

$$\delta_\tau^2 B^{(4)}_{\mu_1\mu_2\mu_3\mu_4} = (\delta_\tau \nabla_{\mu_1}) \tau_{\mu_2\mu_3\mu_4} + \nabla_{\mu_1} \delta_\tau \tau_{\mu_2\mu_3\mu_4} + \text{cycl.} \tag{A.4}$$

The first term gives

$$\begin{aligned}
&(\delta_\tau \nabla_{\mu_1}) \tau_{\mu_2\mu_3\mu_4} = \\
&-6\,(3a_3 + 2a_{10})\, \nabla_{\mu_1} \nabla^\alpha R \tau_{\mu_2\mu_3}{}^\beta \tau_{\mu_4\alpha\beta} - 6\,(3a_3 + 2a_{10})\, \nabla^\alpha R \tau_{\mu_1\mu_2}{}^\beta \nabla_{\mu_3} \tau_{\mu_4\alpha\beta}
\end{aligned} \tag{A.5}$$

$$
\begin{aligned}
&+(15a_3+6a_7)\nabla^\beta R\tau_{\mu_1\mu_2\beta}\nabla^\alpha\tau_{\mu_3\mu_4\alpha}-6(5a_3+2a_7)\nabla_{\mu_1}R\tau_{\mu_2\mu_3}{}^\alpha\nabla^\beta\tau_{\mu_4\alpha\beta}\\
&+(9a_3+6a_{10})\nabla^\alpha R\tau_{\mu_1\mu_2}{}^\beta\nabla_\beta\tau_{\mu_3\mu_4\alpha}-6(5a_3+2a_7)R\tau_{\mu_1\mu_2}{}^\alpha\nabla_{\mu_3}\nabla^\beta\tau_{\mu_4\alpha\beta}\\
&+3a_3R\tau_{\mu_1\mu_2\mu_3}\nabla^\alpha\nabla^\beta\tau_{\mu_4\alpha\beta}-3a_3g_{\mu_1\mu_2}R\tau_{\mu_3\mu_4}{}^\alpha\nabla^\beta\nabla^\gamma\tau_{\alpha\beta\gamma}\\
&+(15a_3+6a_7)R\tau_{\mu_1\mu_2}{}^\alpha\nabla_\alpha\nabla^\beta\tau_{\mu_3\mu_4\beta}-6a_3\tau_{\mu_1\mu_2}{}^\alpha\nabla_{\mu_3}\nabla_{\mu_4}\nabla^\beta\nabla^\gamma\tau_{\alpha\beta\gamma}\\
&-12a_4\tau_{\mu_1\mu_2}{}^\alpha\nabla_{\mu_3}\nabla^\beta\nabla_\beta\nabla^\gamma\tau_{\mu_4\alpha\gamma}-12a_5\tau_{\mu_1\mu_2\mu_3}\nabla_{\mu_4}\nabla^\alpha\nabla^\beta\nabla^\gamma\tau_{\alpha\beta\gamma}\\
&+6a_4\tau_{\mu_1\mu_2}{}^\alpha\nabla_\alpha\nabla^\beta\nabla_\beta\nabla^\gamma\tau_{\mu_3\mu_4\gamma}+6a_5g_{\mu_1\mu_2}\tau_{\mu_3\mu_4}{}^\alpha\nabla_\alpha\nabla^\beta\nabla^\gamma\nabla^\delta\tau_{\beta\gamma\delta}\\
&+\frac{3}{2}(3a_2-a_3+2a_8)R^2\tau_{\mu_1\mu_2\mu_3}\tau_{\mu_4}{}^\alpha{}_\alpha+(3a_3-3a_2)\nabla_{\mu_1}\nabla^\beta R\tau_{\mu_2\mu_3\beta}\tau_{\mu_4}{}^\alpha{}_\alpha\\
&+3(a_2-a_3)\nabla^\beta\nabla_{\mu_1}R\tau_{\mu_2\mu_3\beta}\tau_{\mu_4}{}^\alpha{}_\alpha+6(a_2+a_3-2a_{12})\nabla_{\mu_1}\nabla^\alpha R\tau_{\mu_2\mu_3\mu_4}\tau_\alpha{}^\beta{}_\beta\\
&+\frac{3}{2}(-3a_2+a_3-2a_8)g_{\mu_1\mu_2}R^2\tau_{\mu_3\mu_4}{}^\alpha\tau_\alpha{}^\beta{}_\beta-3(a_2-a_3+2a_{11})\nabla_{\mu_1}\nabla_{\mu_2}R\tau_{\mu_3\mu_4}{}^\alpha\tau_\alpha{}^\beta{}_\beta\\
&-3(a_2+a_3-2a_{12})g_{\mu_1\mu_2}\nabla^\alpha\nabla^\beta R\tau_{\mu_3\mu_4\alpha}\tau_\beta{}^\gamma{}_\gamma+6(a_2+a_8-a_{11})\nabla^\beta R\tau_{\mu_1\mu_2\beta}\nabla_{\mu_3}\tau_{\mu_4}{}^\alpha{}_\alpha\\
&+6(a_2+a_3-2a_{12})\nabla^\alpha R\tau_{\mu_1\mu_2\mu_3}\nabla_{\mu_4}\tau_\alpha{}^\beta{}_\beta-6(2a_2-a_3+a_8+a_{11})\nabla_{\mu_1}R\tau_{\mu_2\mu_3}{}^\alpha\nabla_{\mu_4}\tau_\alpha{}^\beta{}_\beta\\
&+12(a_2+a_3-a_9)\nabla_{\mu_1}R\tau_{\mu_2\mu_3\mu_4}\nabla^\alpha\tau_\alpha{}^\beta{}_\beta-6(a_2+a_3-a_9)g_{\mu_1\mu_2}\nabla^\gamma R\tau_{\mu_3\mu_4\gamma}\nabla^\alpha\tau_\alpha{}^\beta{}_\beta\\
&-6(a_2+a_8-a_{11})\nabla_{\mu_1}R\tau_{\mu_2\mu_3}{}^\alpha\nabla_\alpha\tau_{\mu_4}{}^\beta{}_\beta-3(a_2+a_3-2a_{12})g_{\mu_1\mu_2}\nabla^\alpha R\tau_{\mu_3\mu_4}{}^\beta\nabla_\beta\tau_\alpha{}^\gamma{}_\gamma\\
&+3(-3a_2+a_3-2a_8)R\tau_{\mu_1\mu_2}{}^\alpha\nabla_{\mu_3}\nabla_{\mu_4}\tau_\alpha{}^\beta{}_\beta+6a_6g_{\mu_1\mu_2}\tau_{\mu_3\mu_4}{}^\alpha\nabla_\alpha\nabla^\beta\nabla_\beta\nabla^\gamma\tau_\gamma{}^\delta{}_\delta\\
&+3a_2R\tau_{\mu_1\mu_2\mu_3}\nabla^\alpha\nabla_\alpha\tau_{\mu_4}{}^\beta{}_\beta-3a_2g_{\mu_1\mu_2}R\tau_{\mu_3\mu_4}{}^\alpha\nabla^\beta\nabla_\beta\tau_\alpha{}^\gamma{}_\gamma\\
&-3(a_1+2(a_2+a_3-a_9))g_{\mu_1\mu_2}R\tau_{\mu_3\mu_4}{}^\alpha\nabla_\alpha\nabla^\beta\tau_\beta{}^\gamma{}_\gamma-6a_2\tau_{\mu_1\mu_2}{}^\alpha\nabla_{\mu_3}\nabla_{\mu_4}\nabla^\beta\nabla_\beta\tau_\alpha{}^\gamma{}_\gamma\\
&-6a_1\tau_{\mu_1\mu_2}{}^\alpha\nabla_{\mu_3}\nabla_{\mu_4}\nabla_\alpha\nabla^\beta\tau_\beta{}^\gamma{}_\gamma-12a_6\tau_{\mu_1\mu_2\mu_3}\nabla_{\mu_4}\nabla^\alpha\nabla_\alpha\nabla^\beta\tau_\beta{}^\gamma{}_\gamma\\
&+3(a_1+4(a_2+a_3-a_9))R\tau_{\mu_1\mu_2\mu_3}\nabla_{\mu_4}\nabla^\alpha\tau_\alpha{}^\beta{}_\beta
\end{aligned}
$$

with the symmetrization understood on both sides of the equation. These terms need to be canceled by the r.h.s. in (A.4)

$$\nabla_{\mu_1}\delta_\tau\tau_{\mu_2\mu_3\mu_4} \tag{A.6}$$

It follows that the coefficients in front of all the terms in (A.5) which do not contain any ∇_{μ_i} must be zero. This gives a system of equations for the coefficients a_i with a solution that all a_i are zero except a_{11}. Now we argue that, also, a_{11} must be zero. The variation δ_τ of metric reads

$$\delta_\tau g_{\mu\nu}=a_{11}\nabla_\mu R\tau_\nu{}^\alpha{}_\alpha \tag{A.7}$$

Then, $\delta^2_\tau B_{\mu_1\mu_2\mu_3\mu_4}$ is reduced to

$$
\begin{aligned}
\delta^2_\tau B^{(4)}_{\mu_1\mu_2\mu_3\mu_4}=4\nabla_{\mu_1}\delta_\tau\tau_{\mu_2\mu_3\mu_4}\ &-24\ a_{11}\nabla_{\mu_1}\nabla_{\mu_2}R\tau_{\mu_3\mu_4}{}^\alpha\tau_\alpha{}^\beta{}_\beta\\
&-24\ a_{11}\nabla^\beta R\tau_{\mu_1\mu_2\beta}\nabla_{\mu_3}\tau_{\mu_4}{}^\alpha{}_\alpha\\
&-24\ a_{11}\nabla_{\mu_1}R\tau_{\mu_2\mu_3}{}^\alpha\nabla_{\mu_4}\tau_\alpha{}^\beta{}_\beta\\
&+24\ a_{11}\nabla_{\mu_1}R\tau_{\mu_2\mu_3}{}^\alpha\nabla_\alpha\tau_{\mu_4}{}^\beta{}_\beta
\end{aligned}
\tag{A.8}
$$

where symmetrization in μ_i is understood on the right hand side. Note that in each term in (A.8) there is a τ with two external indices without any derivative acting on it. So, we

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conclude that a_{11} must be zero because whatever choice we take for $\delta_\tau \tau_{\mu_2\mu_3\mu_4}$, the term $\nabla_{\mu_1}\delta_\tau\tau_{\mu_2\mu_3\mu_4}$ will not be able to cancel the last four terms in (A.8).

In summary, we have shown that

$$\delta_\tau g_{\mu\nu} = 0 \tag{A.9}$$

no matter what $\delta_\tau \tau_{\mu_2\mu_3\mu_4}$ is. In turn, eq. (A.9), together with eq. (A.4), implies that

$$\delta_\tau \tau_{\mu\nu\rho} = 0 \tag{A.10}$$

To conclude this appendix, let us justify the claim we made that eq. (5.26) is the most general $2n$-th order covariant cocycle. To this end we prove that terms of the form

$$\omega^{\mu_1\ldots\mu_{2n-1}}\,\epsilon_{\mu_1\alpha_1}\,\epsilon_{\mu_2\alpha_2}\ldots\epsilon_{\mu_k\alpha_k}\,\nabla^{\alpha_1}\nabla^{\alpha_2}\ldots\nabla^{\alpha_k}\nabla_{\mu_{k+1}}\ldots\nabla_{\mu_{2n-1}}R \tag{A.11}$$

are in fact *equivalent* to either I^ω or K^ω. Using the formula valid in $D=2$,

$$\epsilon_{\alpha\beta}\epsilon_{\mu\nu} = g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu} \tag{A.12}$$

we can eliminate the ϵ-tensors two by two in (A.11). In every step we produce two terms, out of which the first is zero because of the tracelessness of ω and the second contracts two indices of ω with the indices of the covariant derivatives on the right side. The form of the final expression will depend on parity of k; in the case of even k we get $(-1)^{k/2}\,I^\omega$ and in the case of odd k, $(-1)^{(k-1)/2}\,K^\omega$. Therefore, all such terms are already included in the general form of Δ_ω, eq. (5.26).

B. Diff cocycles

This appendix is devoted to eqs. (5.9) and (5.10). Our result is that while (5.9) has a nontrivial solution, the solutions to eq. (5.10) are all trivial. The solution to (5.9) is the well-known diffeomeorphim anomaly in 2d, therefore we will concentrate on (5.10). Although the same method could be easily applied to find the explicit solutions to (5.9), we will not do it here. In the sequel of this appendix the terms covariance and covariant are used with reference to diffeomorphisms.

The basic idea of this appendix is to apply the results of [65] by adapting them to the present case. First of all let us notice that the anomaly analysis is carried out in the Euclidean. We will denote Euclidean tensor indices by lower case Latin letters.

Let us start from a general result in [65]. The general form of the solutions to (5.10) is

$$\Delta_\xi^{(4)} = \int d^2x\,(\partial_m\xi^m\,\mathfrak{b} + \partial_{p_1}\partial_{p_2}\xi^m\mathfrak{b}_m^{p_1p_2}) \tag{B.1}$$

where $\mathfrak{b}$ and $\mathfrak{b}^m_{p_1p_2}$ are polynomial expressions of the fields and their derivatives in which all the indices are saturated except for the explicitly shown ones, and $\mathfrak{b}$ is not itself a derivative. Notice that $\mathfrak{b}, \mathfrak{b}^m_{p_1p_2}$ are *not*, in general, covariant tensors. For future reference let us call *first type* and *second type* the cocycles having the form of the first and second term in the r.h.s. of (B.1), respectively.

We stress that in this appendix we cannot, in general, use covariance as a classifying device. This obliges us to trudge our way through a multitude of inelegant and unfamiliar formulas.

The first type cocycles were discussed both in [66] and [67]. Any such cocycle[9] is a partner of a Weyl cocycle and can be eliminated in favor of the partner by subtracting a suitable counterterm. Since we have shown in [11] that, at order four, there are no non-trivial Weyl cocycles (trace anomalies), we will disregard these cocycles altogether and concentrate on cocycles of the second type in (B.1), i.e. on cocycles proportional to $\partial_{p_1}\partial_{p_2}\xi^m$. It is easy to realize that $\mathfrak{b}^m_{p_1p_2}$ can be synthetically written in the following general form

$$\mathfrak{b} = A_1 + \Gamma A_2 + \Gamma\Gamma A_3 + \Gamma\Gamma\Gamma A_4 + \partial\Gamma A_5 + \Gamma\partial\Gamma A_6 + \partial\partial\Gamma A_7 \tag{B.2}$$

where we have understood all the indices (for instance A_1 stands for $A_1{}_m^{p_1p_2}$) and $A_1, \dots A_7$ are for weight 1 covariant tensors. The symbol Γ represents the linear (not necessarily metric) connection Γ^l_{nm}.

An important remark is that, since $\Delta^{(4)}_\xi$ is degree 4, it follows that all the expressions A_i can only be linear in the background field $B^{(4)}$ and contain $4-i$ derivatives for $i = 1, 2, 3, 4$, one derivative for $i = 5$ and no derivatives for $i = 6, 7$.

As we said before the fact that we cannot use covariance in expressing $\mathfrak{b}^m_{p_1p_2}$ is a tremendous complication. There are however expedients one can use to simplify one's life. One such contrivance consists in splitting the functional operator δ_ξ into two parts

$$\delta_\xi = \delta^c_\xi + \hat{\delta}_\xi \tag{B.3}$$

where δ^c_ξ acts on cochains as if they where covariant tensors, while $\hat{\delta}_\xi$ represents the non-covariant part of the δ_ξ action.

For instance we have

$$\begin{aligned}
\hat{\delta}_\xi\, \Gamma^l_{mn} &= \partial_m\partial_n\xi^l \\
\hat{\delta}_\xi\, \partial_k\Gamma^l_{mn} &= \partial_k\partial_n\partial_m\xi^l + \partial_k\partial_m\xi^p\Gamma^l_{pn} + \partial_k\partial_n\xi^p\Gamma^l_{pm} - \partial_k\partial_p\xi^l\Gamma^p_{mn} \\
\hat{\delta}_\xi\, \xi^l &= -\xi^n\partial_n\xi^l \\
\hat{\delta}_\xi\, \partial_n\xi^l &= -\partial_n\xi^m\partial_m\xi^l \\
\hat{\delta}_\xi\, \partial_m\partial_n\xi^l &= 0 \\
\hat{\delta}_\xi\, \partial_m\partial_n\partial_p\xi^l &= \partial_m\partial_n\xi^q\, \partial_q\partial_p\xi^l + \partial_p\partial_m\xi^q\, \partial_q\partial_n\xi^l + \partial_n\partial_p\xi^q\, \partial_q\partial_m\xi^l
\end{aligned} \tag{B.4}$$

It is easy to prove that

$$\hat{\delta}^2_\xi = 0,$$

but one must be careful: in general $\hat{\delta}_\xi$ does not commute with the operation of differentiation except when particular conditions are met. The latter include the cases when $\hat{\delta}_\xi$ acts on forms or on expressions without unsaturated indices.

[9] The corresponding anomaly does not contain the $\epsilon_{\mu\nu}$ tensor with an unsaturated index, see eq. (2.2).

It is convenient to write the integrand of (B.1) as a two-form. So we write

$$\Delta_\xi = \int d^2x\, \partial_{p_1}\partial_{p_2}\xi^m \mathfrak{b}_m^{p_1p_2} \equiv \int Q_2^1 \tag{B.5}$$

The lower index in Q_n^i represents the form order, the upper index denotes the ghost number (number of ξ factors). We must have $\delta_\xi\Delta_\xi = \hat{\delta}_\xi\Delta_\xi = 0$. Therefore

$$\hat{\delta}_\xi Q_2^1 = dQ_1^2 \tag{B.6}$$

for some one-form Q_1^2. Applying $\hat{\delta}_\xi$ to both sides of this equation and the local Poincaré lemma,[10] we get

$$\hat{\delta}_\xi Q_1^2 = dQ_0^3 \tag{B.7}$$

and, of course, $\hat{\delta}_\xi Q_0^3 = 0$. The reason why we introduce these descent equations is that the classification problem is easier on lower order forms (with higher ghost number) than on higher order forms. Briefly stated the strategy consist in chopping off as many coboundaries and first type cocycles as possible, so as to be left with a subset of possibilities which can be easily dealt with.

Schematically, first one proves that solutions to $\hat{\delta}_\xi(Q_2^1 - dP_1^1) = 0$, where Q_2^1 is defined by (B.5), either correspond to first type cocycles or are trivial. As a consequence of this one proves that solutions to $\hat{\delta}_\xi(Q_1^2 - dP_0^2) = 0$, where Q_1^2 is defined by (B.6), are trivial. Thus possible non-trivial second type cocycles are to be looked for among the Q_0^3 (defined by (B.7)) that do not vanish (up to a diff transformation), of which none exist.

Let us go now to a more detailed description. We need to introduce some notation. Let ω be a 0,1 or 2-form with component $\omega, \omega_m, \omega_{nm}$, respectively. We define the dual tensor

$$\tilde{\omega}^{nm} = \varepsilon^{nm}\omega,\ \tilde{\omega}^n = \varepsilon^{nm}\omega_m, \qquad \tilde{\omega} = \varepsilon^{nm}\omega_{nm}$$

where ε is the constant antisymmetric symbol. We remark that if ω is an exact 1-form, the corresponding dual tensor is a divergence. Next let us introduce a distinction which is basic in the economy of our proof: we separate all the cochains Q_n^i, P_n^i into two classes, class A and class B. Any term is class A if it contains only $\partial\partial\xi$ or higher derivatives of ξ, it is class B otherwise.

We are now ready to state the first lemma.

Lemma 1. *A cocycle Δ_ξ in (B.5) that satisfies*

$$\hat{\delta}_\xi(Q_2^1 - dP_1^1) = 0 \tag{B.8}$$

is either a first type cocycle or a coboundary. In the latter case P_1^1 can be chosen to be class A.

[10] By local Poincaré lemma we mean a basic property of local field theory: if a p-form is a polynomial made of local fields and their derivatives, whose exterior derivative vanishes, either it is a top form, or it is a constant if it is a 0-form, or it is a total derivative. This is an off-shell statement.

The proof in [65] (Theorem 5.1 there) applies to the present case with obvious modifications. Let us give just one example (out of many). The expression $\Omega_\xi = \sqrt{-g}\partial\partial\xi B\nabla R$ ($B \equiv B^{(4)}$), with indices contracted in all possible ways, is an example of (B.8) where Q^1_2 is given by the dual of Ω_ξ and P^1_1 vanishes. It corresponds in fact to a coboundary generated by the counterterm $\int \sqrt{-g}\Gamma B\nabla R$ where the Γ indices are contracted in the same way as the $\partial\partial\xi$ indices in Ω_ξ.

From Lemma 1 and (B.5), (B.6), it is easy to show that Q^2_1 corresponds to a coboundary Q^1_2 if and only if

$$Q^2_1 = \hat{\delta}_\xi P^1_1 + dP^2_0 \tag{B.9}$$

for some class A (or vanishing) P^1_1 and some ghost number 2 0-form P^2_0.

Another piece of independent information is provided by the following

Lemma 2. *Q^2_1 defined by eq. (B.6) can be written in a class A form, that is in a form bilinear either in $\partial\partial\xi$ or linear in both $\partial\partial\xi$ and $\partial\partial\partial\xi$.*

The proof is as follows. The dual tensor to Q^2_1 can be written in the general form

$$\begin{aligned} &\xi\xi F_1 + \xi\partial\xi F_2 + \xi\partial\partial\xi F_3 + \xi\partial\partial\partial\xi F_4 + \xi\partial\partial\partial\partial\xi F_5 + \xi\partial\partial\partial\partial\partial\xi F_6 + \partial\xi\partial\xi F_7 \\ &+\partial\xi\partial\partial\xi F_8 + \partial\xi\partial\partial\partial\xi F_9 + \partial\xi\partial\partial\partial\partial\xi F_{10} + \partial\partial\xi\partial\partial\xi F_{11} + \partial\partial\xi\partial\partial\partial\xi F_{12} \end{aligned} \tag{B.10}$$

where F_i with $i = 1, \dots, 12$ are in general not tensors: they may contain Γ factors. For simplicity all the indices are understood. For instance $\xi\xi F_1$ stands for $\xi^i\xi^j F_{1ij}{}^l$.

Now one can see that, as a consequence of (B.2), (B.4) and (B.5), dQ^2_1 must be class A. Then, dualizing, we see that, in particular, it must be $\partial_l F_{1ij}{}^l = 0$. This implies, by the local Poincaré lemma, that $F_{1ij}{}^l = \partial_m F_{ij}{}^{lm}$ for a suitable tensor F antisymmetric in l, m. But then we can write

$$\partial_l(\xi^i\xi^j F_{1ij}{}^l) = \partial_m((\partial_l\xi^i\xi^j + \xi^i\partial_l\xi^j)F_{ij}{}^{lm})$$

This means that F_1 can be absorbed into F_2. We can repeat the same trick on the other terms. F_6 and F_{10} do not contain derivatives, therefore they must vanish. All the other terms can be reduced to the form F_{11} and F_{12}. This proves the lemma.

It follows from (B.9) that dP^2_0 is class A. But then, using the same argument as in the previous lemma, it is easy to see that P^2_0 itself is class A.

Next one can prove:

Lemma 3. *Q^2_1, given by eq. (B.6), is a coboundary if and only if*

$$\hat{\delta}_\xi(Q^2_1 - dP^2_0) = 0 \tag{B.11}$$

where P^2_0 is class A or 0.

The *only if* part follows immediately by applying $\hat{\delta}_\xi$ to (B.9) and using the previous remark. The proof of the *if* part is more complicated. We give it here in some detail. We

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have to prove that if Q_1^2 satisfies (B.11) then it can be written in the form (B.9). Let us write down the general form of the components of Q_1^2:

$$Q_1^2: \quad \partial\partial\xi\,\partial\partial\xi(A_l + \Gamma B_l) + \partial\partial\xi\,\partial\partial\partial\xi C_l \tag{B.12}$$

where A_l, B_l, C_l are covariant tensors. All the indices have been understood except the one-form component index l. It it easy to see that dP_0^2, being class A, can be absorbed into Q_1^2; therefore we will not indicate it explicitly. The tensors in (B.12) have evident symmetry properties in the indices which we will not spell out. Acting now with $\hat{\delta}_\xi$ on Q_1^2, after some simple algebra we get:

$$\partial_{p_1}\partial_{p_2}\xi^i\partial_{q_1}\partial_{q_2}\xi^j\left(\partial_{r_1}\partial_{r_2}\xi^k\, B_{lijk}^{p_1p_2q_1q_2r_1r_2} - 3\partial_j\partial_{q_3}\xi^k\, C_{lik}^{p_1p_2q_1q_2q_3}\right)$$

To satisfy (B.11) this must vanish, which implies the constraint

$$B_{lijk}^{p_1p_2q_1q_2r_1r_2} = 3C_{lik}^{p_1p_2q_1q_2r_1}\delta_j^{r_2} \tag{B.13}$$

while the tensor A_l is unconstrained.

Now let us write a class A P_1^1 in the form

$$P_1^1: \quad \partial\partial\xi(\Gamma\, K_l + \Gamma\,\Gamma\, L_l + \partial\Gamma\, M_l) \tag{B.14}$$

with the same conventions as above. There are other possible terms one could add, but these will suffice. After operating with $\hat{\delta}_\xi$ we get

$$\begin{aligned} \hat{\delta}_\xi P_1^1: \quad & -\partial_{p_1}\partial_{p_2}\xi^i\Big[\partial_{q_1}\partial_{q_2}\xi^j\, K_{lij}^{p_1p_2q_1q_2} + \partial_r\partial_{q_1}\partial_{q_2}\xi^j\, M_{lij}^{p_1p_2q_1q_2r} \\ & + \partial_{q_1}\partial_{q_2}\xi^j\Gamma_{r_1r_2}^k\left(L_{lijk}^{p_1p_2q_1q_2r_1r_2} + 2M_{lik}^{p_1p_2q_1r_1q_2}\delta_j^{r_2} - M_{lij}^{p_1p_2r_1r_2q_1}\delta_k^{q_2}\right)\Big] \end{aligned} \tag{B.15}$$

From this we see that $\hat{\delta}_\xi P_1^1$ reproduces Q_1^2 provided

$$\begin{aligned} K_{lij}^{p_1p_2q_1q_2} &= -A_{lij}^{p_1p_2q_1q_2} \\ M_{lij}^{p_1p_2q_1q_2q_3} &= -C_{lij}^{p_1p_2q_1q_2q_3} \\ L_{lijk}^{p_1p_2q_1q_2r_1r_2} &= C_{lik}^{p_1p_2q_1q_2(r_1}\delta_j^{r_2)} + C_{lij}^{p_1p_2r_1r_2(q_1}\delta_k^{q_2)} \end{aligned} \tag{B.16}$$

where indices in round brackets are meant to be symmetrized. We notice that this implies in particular that the tensor M must be chosen symmetric in the q_1, q_2, q_3 indices. This proves Lemma 3.

At this point our quest comes to an end, because from (B.11) and (B.7), it follows that

$$d(Q_0^3 - \hat{\delta}_\xi P_0^2) = 0 \tag{B.17}$$

Therefore, since we want to find possible non-trivial second type cocycles, we have to look among the Q_0^3 that do not satisfy eq. (B.17). We will prove that there exist none. One way to see it is as follows. Since Q_1^2 is class A (lemma 2), also $\hat{\delta}_\xi Q_1^2$ is. Using an argument similar to the one which leads to Lemma 2, from (B.7) one can show that Q_0^3 can be written in the form

$$\partial_{p_1}\partial_{p_2}\xi^i\,\partial_{q_1}\partial_{q_2}\xi^j\,\partial_r\xi^k\,E_{ijk}^{p_1p_2q_1q_2r} \tag{B.18}$$

where E is a tensor antisymmetric in the exchange of the triple $\{p_1, p_2, i\}$ with $\{q_1, q_2, j\}$.

Next, differentiating the E term we get

$$2\partial_s\partial_{p_1}\partial_{p_2}\xi^i\,\partial_{q_1}\partial_{q_2}\xi^j\,\partial_r\xi^k\,E_{ijk}^{p_1p_2q_1q_2r} + \partial_{p_1}\partial_{p_2}\xi^i\,\partial_{q_1}\partial_{q_2}\xi^j\,\partial_s\partial_r\xi^k E_{ijk}^{p_1p_2q_1q_2r}$$
$$+\partial_{p_1}\partial_{p_2}\xi^i\,\partial_{q_1}\partial_{q_2}\xi^j\,\partial_r\xi^k\partial_s E_{ijk}^{p_1p_2q_1q_2r}$$

Since dQ_0^3 must be class A, we must have in particular $\partial_s E = 0$. However E is a tensor linear in the $B^{(4)}$ tensor and does not contain any derivatives. It is evident that $\partial_s E = 0$ cannot be satisfied, unless $E \equiv 0$.

In conclusion there are no nontrivial solutions to (5.10).

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